Global Energy Transfer Model of Microwave Induced Plasma in a Microwave Electrothermal Thruster Resonant Cavity

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Microwave electrothermal thruster (MET) is a type of electric propulsion system being studied for use in in-space propulsion applications of satellites and spacecraft. In microwave electrothermal thrusters, the goal is to convert a microwave resonant cavity to a heating chamber of a propulsion system, and thus heating the propellant to be expelled using free floating microwave induced plasma. Once started, plasma acts as a resistive load, absorbs microwave energy and heats up the gas. This study presents a global model which aims to provide a better understanding of the propellant heating mechanism in the microwave resonant cavity of a MET. Formulation for the determination of the electron density, temperature, the heavy particle temperature in the cavity, and power requirement for various desired thrust levels is presented. Equations are solved using COMSOL Multiphysics equation solver to determine the thruster performance and plasma parameters, and preliminary results are presented.

Nomenclature

 A^* =Nozzle throat area

 C_p =Specific heat at constant pressure

- J =Bessel function of the first kind
- k =Specific heat ratio
- k_b =Boltzmann constant
- k_{exc} =Excitation rate coefficient
- k_{iz} =Ionization rate coefficient
- \dot{m} =Mass flow rate

 M_{ex} =Nozzle exit Mach number

- N =Neutral number density
- n_e =Electron number density
- n_i =Ion number density
- n_0 =Electron number density at the center of the plasma

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p_a	=Ambient pressure					
p_c	=Chamber pressure					
p_{ex}	=Nozzle exit pressure					
P_{loss}	=Power loss					
P_{eloss}	=Power loss due to electron flux					
P_{iloss}	=Power loss due to ion flux					
T_c	=Chamber temperature					
T_{ex}	=Nozzle exit temperature					
T_e	=Electron temperature					
T_i	=Inlet temperature					
T_0	=Stagnation temperature					
U_{exc}	=Excitation energy					
U_{iz}	=Ionization energy					
v_{ex}	=Nozzle exit velocity					
χ_{01}	=First zero of the Bessel function of the first kind					
δ	=Average fraction of energy loss					
Γ_T	=Total flux					
σ_{ei}	=Cross section for electron-ion momentum transfer					
σ_{ea}	=Cross section for electron-neutral momentum transfer					
au	=Thrust					
\forall_p	=Plasma volume					

I. Introduction

SPACE propulsion systems are used for in-space propulsive needs of satellites or spacecraft such as orbit Change or attitude control maneuver required based on specific mission objectives. The main categorization of space propulsion systems are made according to the type of energy utilized for providing the expelled propellant's kinetic energy. In chemical propulsion systems, propellants like hydrazine, ammonia or nitrogen can be expelled directly or after combustion process to obtain thrust. Energy of the propellant, depending on the chamber pressure and temperature, is transferred into kinetic energy in the expansion process through a conventional nozzle. On the other hand, in electrical propulsion systems, electrical energy, supplied from spacecraft power and energy storage systems, is converted into propellant's kinetic energy.

Electric propulsion systems are designed for obtaining higher exhaust velocities while minimizing propellant consumption. These systems can be separated into three basic categories: electrostatic, electromagnetic and electrothermal.



Figure 1. Schematic of a microwave electrothermal thruster

Electrothermal thrusters, which are a subclass of the electric propulsion systems, use electrical energy

to increase the propellant's thermal energy. Energized propellant gas thermal energy is transformed into kinetic energy when it expands through an aerodynamic structure like a nozzle.¹ There are three basic types of electrothermal thrusters; resistojets heat the propellant gas by means of a resistive component, arcjets use a DC arc for heating the propellant, and the third kind employs various forms of electromagnetic radiation for the propellant heating process.

Microwave electrothermal thrusters are the type electrothermal thrusters that employ electromagnetic energy at microwave frequencies to heat the propellant gas. Microwave electrothermal thrusters consist of three basic subsystems; microwave applicator, resonant cavity and nozzle. A simplified schematic of a microwave electrothermal thruster is shown in Figure 1. In MET systems, conversion of microwave energy into propellant's thermal energy is achieved inside a resonant cavity.² Microwave induced free floating plasma is used for the propellant heating.^{3,4}

In a microwave electrothermal thruster, a microwave source generates microwave with desired frequency and power level. Microwave electrothermal thrusters working with various frequency and thrust levels have been produced and tested to date.⁵ Generated microwave is transferred into the cavity with a proper antenna. To date, in experiments two types of microwave generators are mostly used: traveling wave tube amplifiers (TWTA) and magnetrons.⁶ Microwave absorption process requires an applicator, like a resonant cavity or waveguide in which plasma is generated.⁴

The conversion of the microwave heated propellant's thermal energy into kinetic energy is achieved by the use of a nozzle. Nozzle can be designed as converging or converging-diverging as suitable for the objective. Generally, the nozzle is designed as a separate part which is attached to the plate at one end of the cavity.

Transfer of microwave energy into kinetic energy is materialized in three steps;

- Free electrons in the propellant gas accelerate due to the electric field inside the resonant cavity according to the Lorentz force. These energized electrons transfer momentum as a result of collisions with other species (atoms, heavy molecules, electrons etc.) in the gas. This process also results in ionization or excitation of neutral particles.⁶
- After successive nonelastic collisions, electron density in the gas reaches a level of initiating plasma discharge provided commensurate electric field density. Breakdown, which can be thought of as the commencing process of the discharge, begins with the acceleration of free electrons in the gas by means of microwave's electric field according to the Lorentz force. Energized electrons convey part of their energy to neutrals by successive collisions. If this transferred energy is on the level of the ionization energy for the gas atoms, new free electrons will be generated. Electron number density keeps on ascending, provided that adequate electric field is continued. Finally, the process concludes with the electron avalanche that onsets discharge. When the plasma discharge begins, it acts as a resistive load and absorbs microwave energy. Average power absorbed in one microwave cycle can be expressed as;

$$P_{ave} = \frac{n_e q^2 E_0^2}{m} \frac{\nu_m}{\nu_m^2 + \omega^2}$$
(1)

where n_e is the electron number density, q is the electron charge, E_0 is the electric field strength, m is the electron mass, ν_m is the electron momentum collision frequency and ω is the applied microwave frequency. Plasma can be thought of as a heater element that works according to the Ohm's law. Average power is proportional to the square of the electric field.⁷ Plasma discharge is maintained as long as the production rate of electrons outnumber the loss of electrons in recombination processes or wall losses.

• Finally, the propellant gains energy, with the principle of "*Joule Heating*", while swirling around the hot plasma. The heated propellant gas provides thrust by expanding through the nozzle or an orifice.^{4, 6}

The primary loss mechanisms in METs are heat transfer from hot plasma to the walls by radiation, convection and conduction. But in a MET working in TM_{011} mode, the plasma will be formed near the nozzle plate.^{5,7} Gas is expelled as soon as it is heated, so thermal losses to the cavity walls will be small. Expelling of dissociated and ionized particles before recombination results in frozen flow losses. These losses are minimized provided that proper design and operational parameters are chosen. Volume of the plasma can also be changed by adjusting the flow rate and the supplied microwave power. Increasing the applied power while the propellant flow rate (thus the chamber pressure) is kept constant enlarges the volume of plasma. On the other hand, keeping the pressure constant without increasing power causes the contraction of the

plasma volume.⁸ This effect can be used to make dimensions unchanged when increasing power. Besides, parameters like gas pressure and flow characteristics (radial or swirling) can be modified to generate plasma in the desired region.



Figure 2. View of the prototype MET operating in BUSTLab

A global model is developed to better understand the propellant heating mechanism in the microwave resonant cavity. Formulation of the evaluation of the electron density, temperature, the heavy particles temperature in the cavity, and the power requirement for desired thrust levels is presented in the next sections.

II. Global Plasma Model

Microwave electrothermal thruster uses microwave induced plasma to heat the gas as mentioned in the previous section. Experiments conducted to date show that once the plasma discharge begins, microwave induced plasma can be sustained at atmospheric pressure levels. Diamant et al. measured that the discharge is maintained at nearly 200 kPa for He, 250 kPa for N_2 , and 270 kPa for N_2O gases in a MET system which is designed for 2.45 GHz frequency and 1 kW power level.⁹ Thus, microwave thrusters can be modeled as atmospheric pressure discharges. To evaluate the characteristics of an atmospheric plasma system, there are different kinds of methods being used. The plasma properties like species number densities, temperatures are determined in these models. One of the modeling approaches is the two-fluid model in which the plasma is considered to be consisting of two kinds of fluids which are electron fluid and heavy particle fluid.^{10,11} Heavy particle fluid consists of neutrals and ions. Their behaviors are determined by using continuity, momentum and energy equations for each fluid as in fluid mechanics.¹² Besides, source terms for ionization, Lorentz force and Joule heating are added to continuity, momentum and energy equations, respectively. On the other hand, in a global or 0-D model, spatial variations of the plasma properties are not taken into account. Volume averaged properties can be used in calculations. Although these kind of models have limitations, they give a very good understanding of the energy transfer mechanisms in a plasma structure and also allow reasonable computational time.¹³

A. Theory and Assumptions

The model discussed in this study is developed for the analysis of a prototype microwave electrothermal thruster being built and tested at the Bogazici University Space Technologies Laboratory (BUSTLab). This prototype MET, a picture of which is shown in Figure 2, is designed for operating at 2.45 GHz frequency and at power levels of up to 1 kW. The delivered power level can be adjusted at 10 W increments. Argon

gas is used as the propellant. The prototype MET is made of stainless steel and consists of the main parts described in the previous section. Nozzle of the MET is designed to be modular for examining the system characteristics for various nozzle geometries and expansion ratios.

For simplicity, in the model, the nozzle of the system is assumed to be optimally expanded, thus the effective exhaust velocity is taken to be equal to the exhaust velocity. Back pressure of the system can be reduced from the atmospheric pressure down to 10^{-3} torr levels by a mechanical pump. Energy loses of the flow in the nozzle are not taken into consideration. Temperature of the background gas is assumed to be uniform in all plasma sections. In addition, the temperature of ions and neutrals are generally taken to be the same as in two fluid models for atmospheric pressure plasmas.^{14, 15} Since the gas velocity in the plasma filled section of the cavity is very low in comparison to the velocities in the nozzle, the static pressure in the nozzle inlet is considered to be equal to the stagnation pressure. Argon propellant is assumed to behave as an ideal gas, and the specific heat ratio is assumed to be constant.



Figure 3. Schematic of the mass and energy transport for the plasma in the resonant cavity of MET

The deposited power is absorbed by the free electrons in the gas. Then, part of this power is transferred to the heavy particles by collisions.⁴ This energy will be transformed into kinetic energy when the gas is expelled through the converging diverging nozzle. Besides, deposited energy is used for ionization and excitation processes. Also, this part of energy is transferred back to the gas as a result of the recombination and de-excitation processes. Energy gain from recombination and de-excitation is assumed to be not significant when compared with the gain due to the elastic collisions. Another energy loss mechanism is the energy deposited to the walls by the striking electrons and ions. Flow in the cavity and the nozzle is taken to be adiabatic, so losses due to radiation, convection and conduction is not taken into consideration in the model. The species in the plasma are considered to have Maxwellian velocity distribution function. And the plasma is considered to be quasi-neutral.

B. Governing Equations

The model consists of two groups of equations. The first group includes the ideal rocket equations which are derived by assuming that the flow in the nozzle is isentropic. This set of equations are used to determine proper mass flow rate provided the specific chamber conditions. Also, the produced thrust is evaluated in terms of these chamber conditions. In order to define the chamber operation conditions, the chamber temperature and the pressure is set to desired value. In the experiments performed to date with gases like O_2 ,

 N_2 and He, discharge temperature is measured in 2000 to 5500 K range, and the measured chamber pressure values ranged from 100 to 300 kPa.^{9,16,17} In this study, maximum temperature and maximum pressure levels are chosen 4000 K and 250 kPa respectively. In the model, BUSTLab MET nozzle dimensions are used to determine the exit pressure, temperature and Mach number values. Nozzle is assumed to be ideal expanded for the given conditions.

The exit Mach number is evaluated iteratively for a known nozzle exit to throat area ratio, A_{ex}/A^* , by using the relation:¹⁸

$$\frac{A_{ex}}{A^*} = \frac{1}{M_{ex}} \left[\frac{2}{k+1} \left(1 + \frac{k-1}{2} M_{ex}^2 \right) \right]^{\frac{k+1}{2(k-1)}}$$
(2)

where k is the specific heat ratio. A_{ex}, A^*, M_{ex} are exit area, throat area and the exit Mach number, respectively. The stagnation temperature and the stagnation pressure are taken to be equal to the chamber temperature and the pressure because of the low velocities in the chamber. Assuming that the pressure is isentropic, an analytic expression can be written for the exit temperature T_{ex} and the exit pressure p_{ex} in terms of M_{ex} as¹⁸

$$T_c = T_{ex} \left(1 + \frac{k-1}{2} M_{ex}^2 \right) \tag{3}$$

$$P_c = P_{ex} \left(1 + \frac{k-1}{2} M_{ex}^2 \right)^{\frac{k}{k-1}}$$
(4)

After performing the nozzle exit condition calculations for pressure and temperature, the exit velocity can be calculated by using;

$$v_{ex} = \sqrt{\frac{2kRT_c}{k-1} \left[1 - \left(\frac{p_{ex}}{p_c}\right)^{\frac{k-1}{k}}\right]} \tag{5}$$

where R is the specific gas constant. Mass flow rate is a function of the throat area, chamber pressure and temperature and defined as;¹⁸

$$\dot{m} = A^* p_c k \sqrt{\frac{(2/k+1)^{(k+1)/(k-1)}}{kRT_c}}$$
(6)

For an optimum expanded nozzle, the thrust due to the pressure, $A_{ex}(p_{ex} - p_a)$, is neglected and the thrust equation is reduced to;

$$\tau = \dot{m}v_{ex} \tag{7}$$

After evaluating the performance parameters, the plasma parameters are determined to investigate the energy transfer mechanisms of microwave power to the propellant gas. The main mechanism that heats the propellant gas is the momentum transfer between the electrons and the heavy particles by elastic collisions as noted above. The microwave energy, deposited into the resonant cavity part of the thruster, is absorbed by the electrons. The energy balance for the electrons have this power input, P_{dep} . Electrons are energized because of the Lorentz force which results from the electric field inside the cavity. Even though the magnetic field component of the microwave acts on the charged particles, its effect is assumed to be negligible in comparison to the electric field. Accelerated electrons will transfer some of their energy with the heavy particles as a result of elastic collisions. The ions are heated by electron-ion elastic collisions, $n_e n_i \langle \sigma_{ei} v_e \rangle \delta_2^3 k_b T_e$, and the neutrals are heated by electron-neutral elastic collisions, $n_e N \langle \sigma_{ea} v_e \rangle \delta_2^3 k_b T_e$. Although some energy is gained by recombination and de-excitation processes, this is much smaller compared to the energy gained by elastic collisions. This can be formulated as;

$$P_{dep} = \underbrace{\forall_p n_e \left(n_i \left\langle \sigma_{ei} v_e \right\rangle + N \left\langle \sigma_{ea} v_e \right\rangle \right) \delta_2^3 k_b T_e}_{Elastic \ Collision \ Loss} \tag{8}$$

where $\delta = \frac{2m_e}{M}$ is the average fraction of energy loss by the electrons in an elastic collision with a heavy particle. Also, $\langle \sigma_{ei} v_e \rangle$ and $\langle \sigma_{ea} v_e \rangle$ are electron-ion and the electron-neutral rate coefficients for elastic momentum transfer collisions averaged over Maxwellian electron energy distribution and are written as;¹⁹

$$\langle \sigma_{ei} v_e \rangle = 2.91 \times 10^{-12} \frac{ln\Lambda}{T_{eV}^{1.5}} \quad [m^3 s^{-1}]$$
 (9)

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$$\langle \sigma_{ea} v_e \rangle = (0.084 + 0.537T_{eV} + 1.192T_{eV}^2) \times 10^{-14} \quad [m^3 s^{-1}] \tag{10}$$

where $ln\Lambda$ is the Coulomb logarithm;

$$ln\Lambda \simeq ln\left(1.55 \times 10^{13} \sqrt{\frac{T_{eV}^3}{n_e}}\right) \tag{11}$$

On the other hand, the total power needed to reach the desired chamber temperature will be higher than the power deposited to the gas because of the losses to the walls and energy used for inelastic collisions. Because of the potential difference as a result of the plasma sheath formation on the walls, the electrons and ions carry some amount of energy out of the plasma when they strike on the walls of the cavity.²⁰ In addition to this, some amount of the energy is used in ionization and excitation processes. As mentioned above, this energy is recovered by recombination and de-excitation processes. In this model, frozen flow losses are neglected because the energy carried by the ions and the excited particles expelled out of the cavity zone are not taken into consideration in the current study. When, the conductive and convective losses are neglected, the total power can be written as;

$$P_{Total} = P_{dep} + P_{loss} + P_{Inelastic \ Collision} \tag{12}$$

Every electron leaving the plasma and striking on the walls carries off an energy of 2 TeV, and for an Argon ion leaving the plasma and striking on the walls, the energy carried away will be 5.2 TeV.²¹ The set of equations for the power lost to the walls can be expressed as;

$$P_{eloss} = 2k_B T_e \Gamma_T \tag{13}$$

$$P_{iloss} = 5.2k_B T_e \Gamma_T \tag{14}$$

$$P_{loss} = P_{eloss} + P_{iloss} \tag{15}$$

where Γ_T is the total ion (or electron) flux from the plasma to the cavity walls.

In our model, there are two main reactions in the plasma zone. These are ionization and excitation processes;

$$e + Ar \to Ar^+ + 2e \tag{16}$$

$$e + Ar \to Ar^* + e \tag{17}$$

The total power for the inelastic collisions is equal to the sum of the excitation and ionization losses;

$$P_{Inelastic \ Collision} = \forall_p \underbrace{(\underbrace{k_{exc}U_{exc}n_eN}_{Excitation} + \underbrace{k_{iz}U_{iz}n_eN}_{Ionization})$$
(18)

where k_{iz} and k_{exc} are the ionization and the excitation rate coefficients, respectively. Curve fits for k_{iz} and k_{exc} are taken from Lieberman et al.;²¹

$$k_{iz} = 2.34 \times 10^{-14} (T_{eV})^{0.59} exp(\frac{-17.44}{T_{eV}}) \quad [m^3 s^{-1}]$$
⁽¹⁹⁾

$$k_{exc} = 2.48 \times 10^{-14} (T_{eV})^{0.33} exp(\frac{-12.78}{T_{eV}}) \quad [m^3 s^{-1}]$$
⁽²⁰⁾

If the recombination is neglected, the electron continuity equation for steady state can be expressed as

$$\nabla^2 n + \frac{\nu_{iz}}{D} n = 0 \tag{21}$$

where the ν_{iz} is the ionization frequency.^{12,21} D is the diffusion coefficient which is assumed to be equal to the ambipolar diffusion coefficient, the form of which is adapted form Jonkers et al.¹⁹ as;

$$D = D_i \left(1 + \frac{T_e}{T_c} \right) \tag{22}$$

Joint Conference of 30th ISTS, 34th IEPC and 6th NSAT, Hyogo-Kobe, Japan July 4–10, 2015 where D_i is the ion diffusion coefficient and is a function of chamber pressure and temperature. It can be written as;¹⁹

$$D_i = \frac{3k_b^2 T_c^2}{8p_c M \Omega(T_c)} \tag{23}$$

The Ω coefficient is calculated in the same way as Jonkers et al.¹⁹

When the continuity equation, Equation 21, is solved for axisymmetric cylindrical cavity, distribution of the electron number density can be evaluated as;²¹

$$n(r,z) = n_0 J_0\left(\chi_{01} \frac{r}{R}\right) \cos\left(\frac{\pi z}{L}\right)$$
(24)

where n_0 , J_0 and the χ_{01} are electron number density at the center of the plasma, zeroth-order Bessel function of the first kind and the first zero of the J_0 , respectively. Species' fluxes can be defined at the two ends of the cavity²¹ as;

$$\Gamma_z = -D\frac{\partial n}{\partial z} = D\frac{\pi}{L}n_0 J_0\left(\chi_{01}\frac{r}{R}\right)$$
(25)

and at the radial wall, where r = R, as;

$$\Gamma_r = -D\frac{\partial n}{\partial r} = D\frac{\chi_{01}}{R} n_0 J_1\left(\chi_{01}\right) \cos\left(\frac{\pi z}{L}\right)$$
(26)

The total flux for the cylindrical volume can be evaluated by integrating the above flux equations for all cavity surfaces;

$$\Gamma_T = 2 \int_0^{2\pi} \int_0^R \Gamma_z(r) r dr d\theta + \int_0^{2\pi} \int_{-L/2}^{L/2} \Gamma_r(z) R dz d\theta$$
(27)

$$\Gamma_T = \frac{4D\pi^2 R^2 n_0 J_1(\chi_{01})}{\chi_{01}L} + 4DL\chi_{01} n_0 J_1(\chi_{01})$$
(28)

Volume averaged electron number density could be calculated by dividing the volume integral of n(r, z) with the total plasma volume as;

$$n_e = \frac{\int_0^{2\pi} \int_0^R \int_{-L/2}^{L/2} n(r, z) r dz dr d\theta}{\forall_p}$$
(29)

$$n_e = \frac{4LR^2 n_0 J_1(\chi_{01})}{\chi_{01} \forall_p} \tag{30}$$

To maintain quasineutrality and continuity, the total volume ionization inside the plasma zone is set equal to the total ion loss out of the plasma zone. Losses because of the diffusion is dominate the losses due to three body recombination in atmospheric pressure discharges.¹⁹ Thus, the recombination losses are neglected. On the other hand, diffusion term is included in the total flux equation. The total flux is balanced with the total generation as;

$$\forall_p n_e N k_{iz} = \Gamma_T \tag{31}$$

By plugging in the expressions obtained for Γ_T in Equation 28, and n_e in Equation 30 into Equation 31, the following relation is obtained;

$$k_{iz} = \frac{D}{N} \left(\frac{\pi^2}{L^2} + \frac{\chi_{01}^2}{R^2} \right)$$
(32)

The neutral density can be obtained by using Dalton's law as;

$$N = \frac{p_c}{k_B T_c} - n_e \left(1 + \frac{T_e}{T_c}\right) \tag{33}$$

III. Results and Discussions

Two sets of equations are solved to determine the thruster performance parameters and plasma parameters in the cavity by using COMSOL Multiphysics equation solver. The first equation set includes equations 2 to 7. In the model, chamber pressure, chamber temperature, nozzle throat area and nozzle exit to throat area ratio are assigned. Since the measured pressure and temperature values for Argon is not available in the literature, the average value of data for Nitrogen and Helium is used.^{16,17} Calculations are performed for 100, 200 and 300 kPa chamber pressure values. Exit Mach number, exit temperature, exit pressure, exit velocity, mass flow rate, thrust values are calculated to determine the thruster operating conditions. The calculated values are presented in Table 1. As seen form the tabulated values, the thrust level increases with increased chamber temperature and pressure values.

$p_c \left[kPa \right]$	$T_c\left[K ight]$	$T_{ex}\left[K ight]$	$p_{ex}\left[Pa\right]$	$v_{ex}\left[m/s ight]$	$\dot{m} \left[mg/s ight]$	$\tau \left[mN \right]$	$n_e [10^{19}/m^3]$	$T_e \left[eV \right]$
100	2000	262	1,93	1456	65	93	1.05	0.7050
150	3000	40	2,89	1760	79	140	1.27	0.7046
200	4000	53	3,87	2032	92	187	1.55	0.7048

 Table 1. System Comparison Table

Steady state solution of electron continuity equation for cylindrical volume is used in the model to evaluate the volume average electron number density. According to this solution, maximum plasma density will be at the center of the cavity. But, experiments conducted to date show that the plasma will be generated at the place of the maximum electric field intensity.^{4,16} But in a global model, place of the plasma is not taken into consideration and it is proper to use volume average of plasma density. To evaluate plasma parameters the second set of equations, which includes the particle and energy balance equations for electrons, are evaluated. The calculated electron number density and electron temperature values are presented in Table 1. As seen from the tabulated values, for the presented chamber pressure and temperature levels, electron number density is on the order of 10^{19} and electron temperature values are around 0.7 eV.

IV. Conclusion

Formulation of equations for the investigation of the power absorption mechanism in a microwave electrothermal thruster resonant cavity is presented. Thruster performance and resonant cavity plasma parameters are evaluated for different chamber temperature and pressure values for a given microwave power level using COMSOL Multiphysics equation solver. The obtained results for the plasma parameters such as electron number density and electron temperature, and the thruster operational parameters such as exit Mach number, exit temperature, exit pressure, exit velocity, mass flow rate and thrust are presented.

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